HEAT TRANSFER FROM VERTICALLY FINNED PLATES BY NATURAL CONVECTION

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Equations are derived for calculating mean and local heat transfer coefficients for vertically finned plates under natural convection. A relation is obtained which will yield the most effective spacing of fins.

Vertically finned plates have found widespread use in the design of various devices for producing the required thermal performance.

Several studies have been made concerning the heat transfer between such surfaces and the surrounding medium. In most of them the heat transfer coefficient was assumed either to be a given quantity [6, 7] or to be determinable on the basis of known formulas for smooth plates [5]. Such studies are useful for exploring various aspects of the heat transfer through finned walls, but they do not yield results from which the heat transfer to the surrounding medium can be calculated with sufficient accuracy.

In order to determine the quantity of heat dissipated from finned surfaces by natural convection, G. N. Dul'nev and N. N. Tarnovskii had proposed a method [1] which served as the basis for subsequent studies [2, 8]. According to this method, the mean heat transfer coefficient is calculated from known formulas for smooth vertical plates, but with a reduced temperature excess between wall and ambient medium taken into account. The ambient temperature is here taken equal to the temperature half-way between fins and is determined from the condition for the development of laminar boundary layers at the fin surfaces.

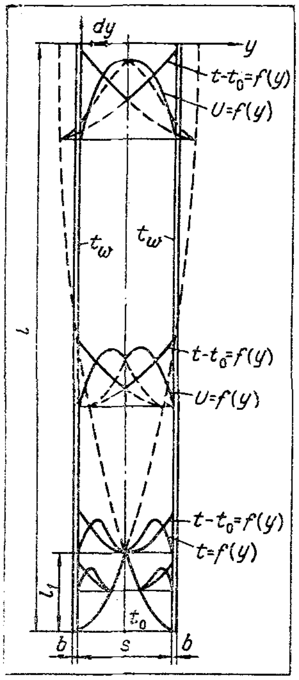

Fig. 1. Assumed model of temperature and velocity distribution curves plotted for the case when air moves between fins.

The Dul'nev–Tarnovskii method allows a better accuracy in the design of plates with vertical rectangular fins. Experimental studies have shown [3, 4], however, that the values of mean heat transfer coefficients in this case still differ considerably from the test values. The reason for this discrepancy may be that no account has been taken of the fact that the pattern of convective flow between fins is different than in the case of a smooth vertical plate. All this makes it necessary to further refine the methods of designing such surfaces.

We will outline here the calculation of vertically finned plates in air. One determines the quantity of heat necessary for raising the temperature of the air which passes between fins from the ambient level to the mean final level. The following assumptions are made in the derivation of the design formulas:

1) The temperature of the fin surfaces is constant and equal to the temperature of the base plate.

2) The distribution of velocities and temperatures during the flow of air between fins is determined by the same relation as in the case of a flat plate surrounded with a laminar boundary layer. All parameters of the boundary layer at a fin surface vary till they reach the values corresponding to $y = s/2$. The proposed model of

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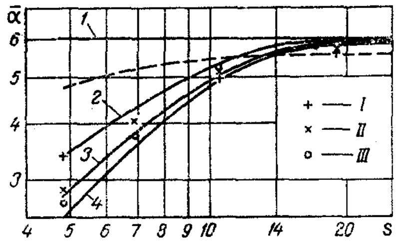

Fig. 2. Mean heat transfer coefficient for a plate, as a function of the distance between fins and of the fin height: based on Eq. (13) (solid curves) at fin height 0 (1), 6.35 mm (2), 12.7 mm (3), 19 mm (4); based on the Dul'nev–Tarnovskii method (dashed curve); test points based on data in [4] for fin height 6.35 mm (I), 12.7 mm (II), 19 mm (III). Heat transfer coefficient $\bar{\alpha}$ (W/m^2 • °C), distance s (mm).

temperature and velocity distribution curves plotted for the case when air moves between fins is shown in Fig. 1. The dashed curves, which have been drawn in addition to the solid ones, indicate the trend which the latter would follow if laminar layers characteristic of smooth vertical plates would develop at the fins. Portions of the curves from the fin surfaces to half-way between them, for $l > l_1$, are shown in solid lines and represent the distribution of velocities and temperatures in accordance with the original premise – with air moving between fins.

The shape of velocity and temperature profiles along the plate height from 0 to l_1 is the one characteristic of smooth vertical plates surrounded with a laminar boundary layer.

3) The heat transfer between the fin ends and the base plate surface separating the fins is determined according to formulas for smooth surfaces. The actual heat transfer may in this case differ somewhat from the calculated one. The error cannot be appreciable, however, because the difference here is in this case an insignificant fraction of the total heat transfer from the entire finned surface. At the same time, these assumptions simplify the calculations considerably.

The quantity of heat which the air receives from the fins can be determined by considering the thermal flux emitted by the fins. The quantity of heat per fin element one unit high, 1 × dy (Fig. 1), is

$$dQ_p = (t - t_0)\, uc\gamma dy. \tag{1}$$

According to [9, 10], we assume the temperature distribution curve for the boundary layer to be

$$t = t_0 + \theta \left(1 - \frac{y}{\delta}\right)^2, \tag{2}$$

and the velocity variation to follow the relation

$$u = 4{,}02\, (g\beta\theta l)^{1/2}\, \frac{y}{\delta} \left(1 - \frac{y}{\delta}\right)^2. \tag{3}$$

The thickness of the boundary layer in these equation is

$$\delta = 5{,}3 \left(\frac{l\nu^2}{g\beta\theta}\right)^{1/4}. \tag{4}$$

The density of air can be said to change with the temperature according to the following relation for the 0-150°C range:

$$\gamma = (1{,}263 - 0{,}00335t)\, 9{,}81. \tag{5}$$

On the basis of Eqs. (2)-(5), the quantity of heat emitted by two lateral fin surfaces is

$$Q_f = 2 \int\limits_0^{s/2} dQ_f = 10{,}6\, (\beta\theta^3)^{1/2} \left(t_1 l^{1/4} s^2 - t_2 s^3 + t_3 \frac{s^4}{l^{1/4}} - t_4 \frac{s^5}{l^{1/2}} + t_5 \frac{s^6}{l^{3/4}} + t_6 \frac{s^7}{l} - t_7 \frac{s^8}{l^{5/4}} \right), \tag{6}$$

where the following designations have been made for clarity:

$$t_1 = \frac{377 - t_w}{\delta_1};\quad t_2 = \frac{377 - 2t_w - \theta}{\delta_1^2} \cdot \frac{2}{3};\quad t_3 = \frac{754 - 2t_w - 3\theta}{\delta_1^3} \cdot \frac{3}{8};$$

$$t_4 = \frac{377 - t_w - 4\theta}{5\delta_1^4};\quad t_5 = \frac{377 - t_w - 14\theta}{48\delta_1^5};\quad t_6 = \frac{3\theta}{56\delta_1^6};$$

$$t_7 = \frac{\theta}{256\delta_1^7};\quad \delta_1 = 5{,}3 \left(\frac{\nu^2}{g\beta\theta}\right)^{1/4}.$$

The thermal flux dissipated along one vertical groove of a finned plate is, with the fin height h,

$$Q = Q_f h + \bar{\alpha}_{sm}\, l\, (b + s)\, \theta. \tag{7}$$

The heat transfer coefficient for a smooth plate $\bar{\alpha}_{sm}$ can be determined from the critical equation [9, 10]

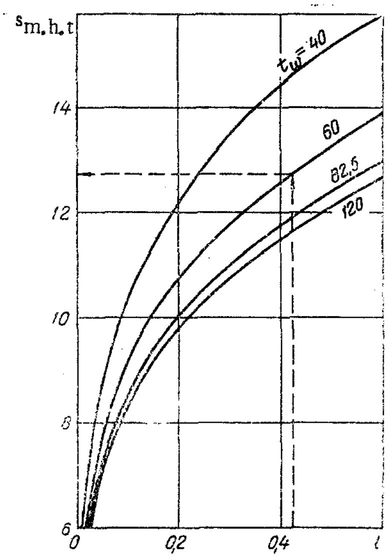

Fig. 3. Graph for determining the distance between fins which corresponds to the maximum heat transfer from a plate: $s_{m.h.t.}$ (mm), l (m), t_w (°C).

$$\overline{\mathrm{Nu}} = 0{,}678 \frac{\mathrm{Pr}^{1/2}}{(0{,}952+\mathrm{Pr})^{1/4}} \mathrm{Gr}^{1/4}. \tag{8}$$

From here

$$\bar{\alpha}_{\mathrm{sm}} = 0{,}678 \frac{\mathrm{Pr}^{1/2}}{(0{,}952+\mathrm{Pr})^{1/4}} \lambda \left(\frac{g\beta\theta}{\nu^2 l}\right)^{1/4}. \tag{9}$$

Considering that the Prandtl number and the ratio $\lambda/\nu^{1/2}$ are almost invariable for air ($\lambda/\nu^{1/2} \approx 6.65$ at normal pressure), we have for the heat transfer from a smooth plate

$$\bar{\alpha}_{\mathrm{sm}} = 5{,}94 \left(\frac{\theta\beta}{l}\right)^{1/4}. \tag{10}$$

Now Eq. (7) becomes

$$Q = Q_f h + 5{,}94 \left(\frac{\theta\beta}{l}\right)^{1/4} l\,(b+s)\,\theta. \tag{11}$$

On the other hand,

$$Q = \bar{\alpha} l\,(2h+b+s)\,\theta. \tag{12}$$

Equating (11) and (12), we obtain, after a few transformations, the following expression for the heat transfer coefficient for a finned plate

$$\bar{\alpha} = \left[10{,}6\,(\beta\theta)^{1/2} h \left(t_1 \frac{s^2}{l^{3/4}} - t_2 \frac{s^3}{l} + t_3 \frac{s^4}{l^{5/4}} - t_4 \frac{s^5}{l^{3/2}} + t_5 \frac{s^6}{l^{7/4}} + t_6 \frac{s^7}{l^2} - t_7 \frac{s^8}{l^{9/4}}\right) + 5{,}94 \left(\frac{\beta\theta}{l}\right)^{1/4} (b+s)\right] / (2h+s+b). \tag{13}$$

The values of the physical parameters in this equation must correspond to the mean air temperature in the boundary layer:

$$\bar{t} = t_0 + \frac{\int_0^\delta u\,(t-t_0)\,c\gamma dy}{\int_0^\delta u c\gamma dy} = t_0 + \frac{\theta\,(377 - t_w + 0{,}464\theta)}{943 - 2{,}49 t_w + 1{,}5\theta}. \tag{14}$$

The values of mean heat transfer coefficients calculated according to Eq. (13) are compared in Fig. 2 with values obtained experimentally [4] and with values obtained by the Dul'nev–Tarnovskii method. Calculations and test data are shown for a plate $l = 0.203$ m high at $t_w = 82.5$°C and $\theta = 64.5$°C. As can be seen here, the calculations based on Eq. (13) agree closely with the test data. The test points do not depart from the calculated curve by more than 8%.

It is well known from the studies in [1, 4] that, as the distance between fins decreases, the rate of heat transfer from a plate first increases to a maximum and then decreases. In order to design effective heat exchangers, one must know what distance between fins will yield the maximum rate of heat transfer. The heat dissipated by a finned surface is more conveniently expressed relative to the rate of heat transfer from a smooth vertical plate with an equal front surface and at the same wall temperature. Then Eq. (11) yields

$$\frac{Q}{Q_{\mathrm{sm}}} = 1{,}78\,(\beta\theta)^{1/4} \frac{h}{s+b} \left(t_1 \frac{s^2}{l^{1/2}} - t_2 \frac{s^3}{l^{3/4}} + t_3 \frac{s^4}{l} - t_4 \frac{s^5}{l^{5/4}} + t_5 \frac{s^6}{l^{3/2}} + t_6 \frac{s^7}{l^{7/4}} - t_7 \frac{s^8}{l^2}\right) + 1. \tag{15}$$

Equating the derivative $d(Q/Q_{\mathrm{sm}})/ds$ to zero, we obtain, after a few transformations, a formula for the optimum distance between fins – corresponding to the maximum heat transfer from the plate:

$$t_1(2b+s) - t_2 \frac{3bs+2s^2}{l^{1/4}} + t_3 \frac{4bs^2+3s^3}{l^{1/2}} - t_4 \frac{5bs^3+4s^4}{l^{3/4}}.$$

$$+ t_5 \frac{6bs^4 + 5s^5}{l} + t_6 \frac{7bs^5 + 6s^6}{l^{5/4}} - t_7 \frac{8bs^6 + 7s^7}{l^{3/2}} . \tag{16}$$

The distance between fins can, according to this equation, be determined (at b = 2.5 mm and t_0 = 18°C) from the graph shown in Fig. 3.

It may become necessary in the design of finned plates to determine the local heat transfer coefficients. We then proceed as follows.

The heat dissipated from a vertical groove of a finned plate whose height is x will, according to Eq. (11), be equal to

$$Q_x = Q_f h + 5{,}94\,(\theta\beta x^3)^{1/4}\,(b + s)\,\theta. \tag{17}$$

As the height of a plate is increased by dx, the heat transfer rate increases by

$$dQ_x = (Q_x)'\,dx. \tag{18}$$

On the other hand,

$$dQ_x = \alpha_x \theta\,(2h + b + s)\,dx. \tag{19}$$

Equating (18) and (19), we obtain after a few transformations

$$\alpha_x = \left[2{,}65\,(\beta\theta)^{1/2} h \left(t_1 \frac{s^2}{x^{3/4}} - t_3 \frac{s^4}{x^{5/4}} + t_4 \frac{2s^5}{x^{3/2}} - t_5 \frac{3s^6}{x^{7/4}} \right.\right.$$

$$\left.\left. - t_6 \frac{4s^7}{x^2} + t_7 \frac{5s^8}{x^{9/4}}\right) + 4{,}46 \left(\frac{\theta\beta}{x}\right)^{1/4} (b + s)\right] \Big/ (2h + b + s). \tag{20}$$

The relations obtained for the heat transfer from vertically finned plates yield the least errors:

1) in the design of plates with short fins, where their effectiveness is approximately 100%. When it is necessary to account for the fin effectiveness, one may additionally use the results in [6, 11, 12].

2) In the calculation of mean (or local) heat transfer coefficients for plates with the distance between fins smaller than twice the thickness of the boundary layer at the top (analyzed here) plate section. If $s \geq 2\delta$, the heat transfer can be calculated by conventional formulas for smooth vertical surfaces.

NOTATION

t	is the temperature, °C;
u	is the flow velocity, m/sec;
c	is the specific heat, J/kg • °C;
γ	is the density (weight), N/m^3;
θ	is the temperature difference between wall and surrounding medium, °C;
δ	is the thickness of boundary layer, m;
g	is the acceleration of free fall, m/sec^2;
β	is the thermal volume expansivity, 1/°C;
l	is the plate height;
ν	is the kinematic viscosity;
b	is the fin thickness, m;
s	is the distance between fins, m;
h	is the fin height, m;
Q	is the thermal flux, W;
α	is the heat transfer coefficient, W/m^2 • °C;
λ	is the thermal conductivity, W/m • °C;
Nu	is the Nusselt number;
Pr	is the Prandtl number;
Gr	is the Grashof number.

Subscripts:

w denotes the wall (temperature);
0 denotes the ambient (temperature);
f denotes the lateral fin surfaces;
m.h.t. denotes the maximum heat transfer;
sm denotes the smooth plate;
x denotes at height x.
Bar above a symbol ($\bar{\alpha}$) denotes the mean value.

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